

Analytic Methods in Geomechanics

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Professor Y-H. Pao “geomechanics” study. Earth science students “coupled mechanics in science and engineering” (1979), we would rather consider it a multidisciplinary field between geophysics and civil engineering. Many postgraduate students in geology and engineering geology engaged in geomechanics research area from different universities. Geologists, seismologists, mining engineers, geophysicists, soil mechanics engineers, structural engineers, and civil engineers. Without a proper background in geotechnics or geomechanics, very often they find the theoretical and numerical techniques used in geomechanics journals or books difficult to understand. As a civil engineer by training, with a master's degree in geotechnical engineering under the supervision of Prof. John Roscoe, my Ph.D. was on soil mechanics at Northwestern University, which included classification, and inelastic deformation of soils, and soil mechanics (e.g., geomaterials). As a graduate student, I had to take a number of fundamental courses in geology, geophysics, and soil mechanics. They could take on geomechanics research area without much difficulty because of the fundamental theory on soil mechanics, and you could move quickly.